Ranking Methods in Machine Learning

A Tutorial Introduction

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Example 1: Recommendation Systems
Example 2: Information Retrieval
Example 2: Information Retrieval
Example 2: Information Retrieval
Problem: Millions of structures in a chemical library. How do we identify the most promising ones?

Example 3: Drug Discovery
Human genetics is now at a critical juncture. The molecular methods used successfully to identify the genes underlying rare mendelian syndromes are failing to find the numerous genes causing more common, familial, non-mendelian diseases . . .

With the human genome sequence nearing completion, new opportunities are being presented for unravelling the complex genetic basis of nonmendelian disorders based on large-scale genomewide studies...
Types of Ranking Problems

- Instance Ranking
- Label Ranking
- Subset Ranking
- Rank Aggregation

?
Instance Ranking

doc1 > doc2, 10

doc3 > doc5, 20

...
Label Ranking

doc1
- sports > politics
- health > money
- ...

doc2
- science > sports
- money > politics
- ...
- ...
- ...
Subset Ranking

query 1

doc1 > doc2, doc3 > doc5...

doc2 > doc4, doc11 > doc3...

...
Rank Aggregation

query 1

results of search engine 1

results of search engine 2

... desired ranking

query 2

results of search engine 1

results of search engine 2

... desired ranking

...
Types of Ranking Problems

- Instance Ranking
- Label Ranking
- Subset Ranking
- Rank Aggregation

- This tutorial
Tutorial Road Map

Part I: Theory & Algorithms

- Bipartite Ranking
- $k$-partite Ranking
- Ranking with Real-Valued Labels
- General Instance Ranking
- RankSVM
- RankBoost
- RankNet

Part II: Applications

- Applications to Bioinformatics
- Applications to Drug Discovery
- Subset Ranking and Applications to Information Retrieval

Further Reading & Resources
Part I

Theory & Algorithms

[for Instance Ranking]
Bipartite Ranking

Relevant (+)  
- doc1+  
- doc2+  
- doc3+  
- ...

Irrelevant (-)  
- doc1-  
- doc2-  
- doc3-  
- doc4-  
- ...


Bipartite Ranking

- **Instance space** $X$

- **Input:** Training sample $S = (S_+, S_-)$:
  
  $S_+ = (x_1^+, \ldots, x_m^+) \in X^m$ (positive examples)
  
  $S_- = (x_1^-, \ldots, x_n^-) \in X^n$ (negative examples)

- **Output:** Ranking function $f : X \rightarrow \mathbb{R}$
Bipartite Ranking

- Instance space $X$

- **Input:** Training sample $S = (S_+, S_-)$:
  
  $S_+ = (x_1^+, \ldots, x_m^+) \in X^m$ (positive examples)
  
  $S_- = (x_1^-, \ldots, x_n^-) \in X^n$ (negative examples)

- **Output:** Ranking function $f : X \to \mathbb{R}$

- Expected error: $\text{er}(f) = \mathbb{P}_{(x,x') \sim \mathcal{D}_+ \times \mathcal{D}_-} [f(x) < f(x')]$

- Empirical error: $\hat{\text{er}}_S(f) = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} 1(f(x_i^+) < f(x_j^-))$
Is Bipartite Ranking Different from Binary Classification?

Example 1

![Diagram showing Bipartite Ranking and Binary Classification examples](image-url)
Is Bipartite Ranking Different from Binary Classification?

Example 1

Classification error $= \frac{1}{4}$

Classification error $= \frac{1}{4}$
Is Bipartite Ranking Different from Binary Classification?

Example 1

Example for $f_1$:
- Classification error = $\frac{1}{4}$
- Ranking error = $\frac{1}{4}$

Example for $f_2$:
- Classification error = $\frac{1}{4}$
- Ranking error = $\frac{1}{2}$
Is Bipartite Ranking Different from Binary Classification?

Example 2
Is Bipartite Ranking Different from Binary Classification?

Example 2

Classification error = $\frac{1}{100}$
Is Bipartite Ranking Different from Binary Classification?

Example 2

Classification error = \( \frac{1}{100} \)

Ranking error = \( \frac{1}{2} \)
Bipartite Ranking:
Basic Algorithmic Framework

Minimize a convex upper bound on the empirical ranking error, possibly with some regularization, over some class of ranking functions:

\[
\min_{f \in \mathcal{F}} \left[ \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \ell(f, x_i^+, x_j^-) + \lambda N(f) \right]
\]

where

\[
\ell(f, x_i^+, x_j^-) : \text{convex upper bound on } 1(f(x_i^+) < f(x_j^-))
\]

\[
N(f) : \text{regularizer}
\]

\[
\lambda > 0 : \text{regularization parameter}
\]

\[
\mathcal{F} : \text{class of ranking functions}
\]
Bipartite RankSVM Algorithm

$$\min_{f \in \mathcal{F}_K} \left[ \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \ell_{hinge}(f, x_i^+, x_j^-) + \frac{\lambda}{2} \|f\|_K^2 \right]$$

$$\ell_{hinge}(f, x_i^+, x_j^-) = \left(1 - (f(x_i^+) - f(x_j^-))\right)_+ \quad [u_+ = \max(u, 0)]$$

$$\mathcal{F}_K = \text{reproducing kernel Hilbert space (RKHS)}$$
with kernel function $K$

$$N(f) = \frac{\|f\|_K^2}{2}$$

[Herbrich et al, 2000; Joachims, 2002; Rakotomamonjy, 2004]
Bipartite RankSVM Algorithm

Introducing slack variables and taking the Lagrangian dual results in the following convex quadratic program (QP) over $mn$ variables $\{\alpha_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\}$:

$$\min_{\alpha} \left[ \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{m} \sum_{l=1}^{n} \alpha_{ij} \alpha_{kl} \phi(x_i^+, x_j^-, x_k^+, x_l^-) - \sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_{ij} \right]$$

subject to $0 \leq \alpha_{ij} \leq C$ $\forall i, j$

where

$$\phi(x_i^+, x_j^-, x_k^+, x_l^-) = \left( K(x_i^+, x_k^+) - K(x_i^+, x_l^-) - K(x_j^-, x_k^+) + K(x_j^-, x_l^-) \right)$$

$$C = \frac{1}{\lambda mn}$$

Can be solved using a standard QP solver, or more efficient methods (e.g. Chapelle & Keerthi, 2010).
Bipartite RankBoost Algorithm

\[
\min_{f \in \mathcal{L}(\mathcal{F}_{\text{base}})} \left[ \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \ell_{\text{exp}}(f, x_i^+, x_j^-) \right]
\]

\[
\ell_{\text{exp}}(f, x_i^+, x_j^-) = \exp \left( - (f(x_i^+) - f(x_j^-)) \right)
\]

\[
\mathcal{L}(\mathcal{F}_{\text{base}}) = \text{linear combinations of functions in some base class } \mathcal{F}_{\text{base}}
\]

[Freund et al, 2003]
Bipartite RankBoost Algorithm

Input: \((S_+, S_-) \in X^m \times X^n\).

Initialize: \(D_1(x_i^+, x_j^-) = \frac{1}{mn}\) for all \(i \in \{1, \ldots, m\}, j \in \{1, \ldots, n\}\).

For \(t = 1, \ldots, T\):

- Train weak learner using distribution \(D_t\); get weak ranker \(f_t \in F_{\text{base}}\).
- Choose \(\alpha_t \in \mathbb{R}\).
- Update: \(D_{t+1}(x_i^+, x_j^-) = \frac{1}{Z_t} D_t(x_i^+, x_j^-) \exp \left(-\alpha_t \left( f_t(x_i^+) - f_t(x_j^-) \right) \right)\)
  
  where \(Z_t = \sum_{i=1}^m \sum_{j=1}^n D_t(x_i^+, x_j^-) \exp \left(-\alpha_t \left( f_t(x_i^+) - f_t(x_j^-) \right) \right)\).

Output final ranking: \(f(x) = \sum_{t=1}^T \alpha_t f_t(x)\).
Bipartite RankNet Algorithm

\[ \min_{f \in \mathcal{F}_{\text{neural}}} \left[ {1 \over mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \ell_{\text{logistic}}(f, x_i^+, x_j^-) \right] \]

\[ \ell_{\text{logistic}}(f, x_i^+, x_j^-) = \log \left( 1 + \exp \left( - (f(x_i^+) - f(x_j^-)) \right) \right) \]

\[ \mathcal{F}_{\text{neural}} = \text{functions represented by some class of neural networks} \]

[Burges et al, 2005]
$k$-partite Ranking

Rating $k$

\[ \text{doc}^k_1, \text{doc}^k_2, \text{doc}^k_3, \ldots \]

\[ \vdots \]

Rating 2

\[ \text{doc}^2_1, \text{doc}^2_2, \text{doc}^2_3, \text{doc}^2_4, \ldots \]

Rating 1

\[ \text{doc}^1_1, \text{doc}^1_2, \text{doc}^1_3, \text{doc}^1_4, \ldots \]
$k$-partite Ranking

- **Instance space** $X$

- **Input:** Training sample $S = (S_1, S_2, \ldots, S_k)$:
  
  $S_k = (x^k_1, \ldots, x^k_{n_k}) \in X^{n_k}$  
  (examples of rating $k$)

  $S_2 = (x^2_1, \ldots, x^2_{n_2}) \in X^{n_2}$  
  (examples of rating $2$)

  $S_1 = (x^1_1, \ldots, x^1_{n_1}) \in X^{n_1}$  
  (examples of rating $1$)

- **Output:** Ranking function $f : X \to \mathbb{R}$

- **Empirical error:**

  $$
  \hat{er}_S(f) = \left( \frac{1}{\sum_{1 \leq a < b \leq k} n_an_b} \right) \sum_{1 \leq a < b \leq k} \sum_{i=1}^{n_b} \sum_{j=1}^{n_a} (b - a) \mathbf{1}(f(x^b_i) < f(x^a_j))
  $$
Minimize a convex upper bound on the empirical ranking error, possibly with some regularization, over some class of ranking functions:

\[
\min_{f \in \mathcal{F}} \left[ \left( \frac{1}{\sum_{1 \leq a < b \leq k} n_an_b} \right) \sum_{1 \leq a < b \leq k} \sum_{i=1}^{n_b} \sum_{j=1}^{n_a} \ell(f, x_i^b, x_j^a, (b - a)) + \lambda N(f) \right]
\]

where

\[
\ell(f, x_i^b, x_j^a, (b - a)) : \text{ convex upper bound on } (b - a) \mathbf{1}(f(x_i^b) < f(x_j^a))
\]

\[
N(f) : \text{ regularizer}
\]

\[
\lambda > 0 : \text{ regularization parameter}
\]

\[
\mathcal{F} : \text{ class of ranking functions}
\]
Ranking with Real-Valued Labels

doc1 $y_1$
doc2 $y_2$
doc3 $y_3$

...
Ranking with Real-Valued Labels

► Instance space $X$

► Real-valued labels $Y = \mathbb{R}$

► Input: Training sample $S = ((x_1, y_1), \ldots, (x_m, y_m)) \in (X \times \mathbb{R})^m$

► Output: Ranking function $f : X \rightarrow \mathbb{R}$

► Empirical error:

$$\widehat{\text{err}}_S(f) = \frac{1}{\binom{m}{2}} \sum_{1 \leq i < j \leq m} |y_i - y_j| \cdot 1 \left( (y_i - y_j)(f(x_i) - f(x_j)) < 0 \right)$$
Ranking with Real-Valued Labels: Basic Algorithmic Framework

Minimize a convex upper bound on the empirical ranking error, possibly with some regularization, over some class of ranking functions:

$$\min_{f \in \mathcal{F}} \left[ \frac{1}{m \choose 2} \sum_{1 \leq i < j \leq m} \ell(f, (x_i, y_i), (x_j, y_j)) + \lambda N(f) \right]$$

where

$$\ell(f, (x_i, y_i), (x_j, y_j)) : \text{convex upper bound on}$$

$$|y_i - y_j| \mathbf{1} \left( (y_i - y_j)(f(x_i) - f(x_j)) < 0 \right)$$

$$N(f) : \text{regularizer}$$

$$\lambda > 0 : \text{regularization parameter}$$

$$\mathcal{F} : \text{class of ranking functions}$$
General Instance Ranking

doc1 > doc1', r1

doc2 > doc2', r2

...
General Instance Ranking

- Instance space $X$
- **Input:** Training sample $S = ((x_1, x'_1, r_1), \ldots, (x_m, x'_m, r_m)) \in (X^2 \times \mathbb{R}_+)^m$
- **Output:** Ranking function $f : X \rightarrow \mathbb{R}$
General Instance Ranking

- Instance space $X$

- **Input:** Training sample $S = ((x_1, x'_1, r_1), \ldots, (x_m, x'_m, r_m)) \in (X^2 \times \mathbb{R}_+)^m$

- **Output:** Ranking function $f : X \rightarrow \mathbb{R}$

- Empirical error: $\widehat{er}_S(f) = \frac{1}{m} \sum_{i=1}^{m} r_i \ I(f(x_i) < f(x'_i))$
General Instance Ranking:  
Basic Algorithmic Framework

Minimize a convex upper bound on the empirical ranking error, possibly with some regularization, over some class of ranking functions:

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N(f) = \frac{\|f\|_K^2}{2}
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[Herbrich et al, 2000; Joachims, 2002]
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\ell_{\exp}(f, x_i, x'_i, r_i) = r_i \exp \left( -\left( f(x_i) - f(x'_i) \right) \right)
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[Freund et al, 2003]
General RankNet Algorithm

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\mathcal{F}_{\text{neural}} = \text{functions represented by some class of neural networks}
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[Burges et al, 2005]
Tutorial Road Map

Part I: Theory & Algorithms

- Bipartite Ranking
- k-partite Ranking
- Ranking with Real-Valued Labels
- General Instance Ranking
- RankSVM
- RankBoost
- RankNet

Part II: Applications

- Applications to Bioinformatics
- Applications to Drug Discovery
- Subset Ranking and Applications to Information Retrieval

Further Reading & Resources
Part II
Applications
[and Subset Ranking]
Human genetics is now at a critical juncture. The molecular methods used successfully to identify the genes underlying rare mendelian syndromes are failing to find the numerous genes causing more common, familial, non-mendelian diseases . . .

With the human genome sequence nearing completion, new opportunities are being presented for unravelling the complex genetic basis of nonmendelian disorders based on large-scale genomewide studies...
Identifying Genes Relevant to a Disease Using Microarray Gene Expression Data

Biological samples \((d)\)

Genes \((N)\)

\(N \gg d\)
Identifying Genes Relevant to a Disease Using Microarray Gene Expression Data

\[
\begin{align*}
\text{Genes} & \quad (N) \\
\text{Biological samples} & \quad (d) \\
N & \gg d
\end{align*}
\]
Identifying Genes Relevant to a Disease Using Microarray Gene Expression Data

<table>
<thead>
<tr>
<th>Genes ((N))</th>
<th>Biological samples ((d))</th>
</tr>
</thead>
</table>

\[ N \gg d \]
Identifying Genes Relevant to a Disease Using Microarray Gene Expression Data

\[ N \gg d \]
Identifying Genes Relevant to a Disease
Using Microarray Gene Expression Data

\[ N \gg d \]
Identifying Genes Relevant to a Disease Using Microarray Gene Expression Data

- Biological samples ($d$)
- Genes ($N$)

$N \gg d$
Formulation as a Bipartite Ranking Problem

Relevant

Not relevant
# Microarray Gene Expression Data Sets

[Golub et al, 1999; Alon et al, 1999]

<table>
<thead>
<tr>
<th>Data Set</th>
<th>No. of Genes</th>
<th>No. of Tissue Samples</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leukemia</td>
<td>7129</td>
<td>72</td>
<td>25 AML / 47 ALL</td>
</tr>
<tr>
<td>Colon cancer</td>
<td>2000</td>
<td>62</td>
<td>40 tumor / 22 normal</td>
</tr>
</tbody>
</table>
# Selection of Training Genes

## Leukemia

<table>
<thead>
<tr>
<th>Positive genes: Markers for AML/ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Myeloperoxidase</td>
</tr>
<tr>
<td>CD13</td>
</tr>
<tr>
<td>CD33</td>
</tr>
<tr>
<td>HOXA9 Homeo box A9</td>
</tr>
<tr>
<td>V-myb</td>
</tr>
<tr>
<td>CD19</td>
</tr>
<tr>
<td>CD10 (CALLA)</td>
</tr>
<tr>
<td>TCL1 (T cell leukemia)</td>
</tr>
<tr>
<td>C-myb</td>
</tr>
<tr>
<td>Deoxyhypusine synthase</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Negative genes</th>
</tr>
</thead>
<tbody>
<tr>
<td>157 genes involved in physiological cellular functions</td>
</tr>
</tbody>
</table>

## Colon cancer

<table>
<thead>
<tr>
<th>Positive genes: Markers for colon cancer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phospholipase A2</td>
</tr>
<tr>
<td>Keratin 6 isoform</td>
</tr>
<tr>
<td>PTP-H1</td>
</tr>
<tr>
<td>TF-III A</td>
</tr>
<tr>
<td>V-raf oncogene</td>
</tr>
<tr>
<td>MAPK kinase 1</td>
</tr>
<tr>
<td>CEA</td>
</tr>
<tr>
<td>Oncoprotein 18</td>
</tr>
<tr>
<td>PEP carboxykinase</td>
</tr>
<tr>
<td>ERK kinase 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Negative genes</th>
</tr>
</thead>
<tbody>
<tr>
<td>56 genes involved in physiological cellular functions</td>
</tr>
</tbody>
</table>
Top-Ranking Genes for Leukemia Returned by RankBoost

♦ Known marker; ♦ Potential marker;
■ Known therapeutic target; □ Potential therapeutic target;
× No link found.

<table>
<thead>
<tr>
<th>Gene</th>
<th>Relevance Summary</th>
<th>t-Statistic Rank</th>
<th>Pearson Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>KIAA0220</td>
<td>■</td>
<td>6628</td>
<td>2461</td>
</tr>
<tr>
<td>G-gamma globin</td>
<td>♦</td>
<td>3578</td>
<td>3567</td>
</tr>
<tr>
<td>Delta-globin</td>
<td>♦</td>
<td>3663</td>
<td>3532</td>
</tr>
<tr>
<td>Brain-expressed HHCPA78 homolog</td>
<td>■</td>
<td>6734</td>
<td>2390</td>
</tr>
<tr>
<td>Myeloperoxidase</td>
<td>♦</td>
<td>139</td>
<td>6573</td>
</tr>
<tr>
<td>Disulfide isomerase precursor</td>
<td>■</td>
<td>6650</td>
<td>575</td>
</tr>
<tr>
<td>Nucleophosmin</td>
<td>♦</td>
<td>405</td>
<td>1115</td>
</tr>
<tr>
<td>CD34</td>
<td>♦</td>
<td>6732</td>
<td>643</td>
</tr>
<tr>
<td>Elongation factor-1β</td>
<td>×</td>
<td>4460</td>
<td>3413</td>
</tr>
<tr>
<td>CD24</td>
<td>♦</td>
<td>81</td>
<td>1</td>
</tr>
<tr>
<td>60S ribosomal protein L23</td>
<td>■</td>
<td>1950</td>
<td>73</td>
</tr>
<tr>
<td>5-aminolevulinic acid synthase</td>
<td>■</td>
<td>4750</td>
<td>3351</td>
</tr>
</tbody>
</table>

[Agarwal & Sengupta, 2009]
Biological Validation

KIAA0220

Rn (vs Leukocytes)

Leukocytes  ALL  AML

[Agarwal et al, 2010]
Problem: Millions of structures in a chemical library. How do we identify the most promising ones?
Formulation as a Ranking Problem with Real-Valued Labels

\[ pIC_{50} = 5.6718 \]

\[ pIC_{50} = 8.2991 \]

\[ pIC_{50} = 4.1317 \]

...
## Cheminformatics Data Sets

[Sutherland et al, 2004]

<table>
<thead>
<tr>
<th>Data Set</th>
<th>No. of Compounds</th>
<th>No. of Chemical (2.5D) Descriptors</th>
<th>pIC$_{50}$ Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>DHFR inhibitors</td>
<td>361</td>
<td>70</td>
<td>3.3 – 9.8</td>
</tr>
<tr>
<td>COX2 inhibitors</td>
<td>292</td>
<td>74</td>
<td>4.0 – 9.0</td>
</tr>
</tbody>
</table>
DHFR Results Using RankSVM

2.5D chemical descriptors
Gaussian kernel

<table>
<thead>
<tr>
<th>Training size</th>
<th>SVR</th>
<th>RankSVM</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>0.4755</td>
<td>0.4601</td>
</tr>
<tr>
<td>48</td>
<td>0.3430</td>
<td>0.3509</td>
</tr>
<tr>
<td>72</td>
<td>0.2840</td>
<td>0.2726</td>
</tr>
<tr>
<td>96</td>
<td>0.2483</td>
<td>0.2351</td>
</tr>
<tr>
<td>120</td>
<td>0.2171</td>
<td>0.2121</td>
</tr>
<tr>
<td>144</td>
<td>0.2023</td>
<td>0.2032</td>
</tr>
<tr>
<td>168</td>
<td>0.2019</td>
<td>0.1817</td>
</tr>
<tr>
<td>192</td>
<td>0.1808</td>
<td>0.1749</td>
</tr>
<tr>
<td>216</td>
<td>0.1816</td>
<td>0.1722</td>
</tr>
<tr>
<td>237</td>
<td>0.1714</td>
<td>0.1681</td>
</tr>
</tbody>
</table>

FP2 molecular fingerprints
Tanimoto kernel

<table>
<thead>
<tr>
<th>Training size</th>
<th>SVR</th>
<th>RankSVM</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>0.3793</td>
<td>0.3546</td>
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<tr>
<td>48</td>
<td>0.2905</td>
<td>0.2896</td>
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<tr>
<td>72</td>
<td>0.2517</td>
<td>0.2421</td>
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<td>96</td>
<td>0.2343</td>
<td>0.2201</td>
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<tr>
<td>120</td>
<td>0.2147</td>
<td>0.2052</td>
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<tr>
<td>144</td>
<td>0.2166</td>
<td>0.1988</td>
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<tr>
<td>168</td>
<td>0.2096</td>
<td>0.1966</td>
</tr>
<tr>
<td>192</td>
<td>0.2056</td>
<td>0.1962</td>
</tr>
<tr>
<td>216</td>
<td>0.1907</td>
<td>0.1787</td>
</tr>
<tr>
<td>237</td>
<td>0.1924</td>
<td>0.1798</td>
</tr>
</tbody>
</table>

[Agarwal et al, 2010]
Application to Information Retrieval (IR)

Information - Wikipedia, the free encyclopedia
Information as a concept has many meanings, from everyday usage to technical settings. The concept of information is closely related to notions of...

Information theory - Wikipedia, the free encyclopedia
Information theory is a branch of applied mathematics and electrical engineering involving the quantification of information...

Information Please
Infoplease.com, a free, authoritative, and respected reference for Internet users, provides a comprehensive encyclopedia, almanac, atlas, dictionary...

Local business results for information near Allston, MA - Change location

Federal Reserve Bank: General Information
www.bos.frb.org - (617) 973-3000 - More

Dana-Farber Cancer Institute
www.dana-farber.org - (617) 632-3000 - 95 reviews
Learning to Rank in IR

q1
Learning to Rank in IR

q1
Learning to Rank in IR

q1
rel1
Learning to Rank in IR

```
q1
rel1    q2
```
Learning to Rank in IR
Learning to Rank in IR

q1
rel1  q2
rel2
Learning to Rank in IR
Learning to Rank in IR

q1
rel1
q2
rel2
q3
Learning to Rank in IR
Learning to Rank in IR

q1
rel1
q2
rel2
q3
rel3

new query
Google Search  I'm Feeling Lucky

qnew
General Subset Ranking

query 1

doc1 > doc2, doc3 > doc5, ...

query 2

doc2 > doc4, doc11 > doc3, ...

...
-General Subset Ranking

- Query space $Q$
- Document space $D$
- Query-document feature mapping $\phi : Q \times D \rightarrow \mathbb{R}^d$
- Input: Training sample $S = (S^1, \ldots, S^m)$:
  
  $$S^i = ((\phi_1^i, \phi_1^i'), \ldots, (\phi_{n_i}^i, \phi_{n_i}^i')) \in (\mathbb{R}^d \times \mathbb{R}^d)^{n_i}$$

  where
  
  $$\phi_j^i = \phi(q^i, d_j^i), \quad \phi_{j'}^i = \phi(q^i, d_{j'}^i)$$

- Output: Ranking function $f : \mathbb{R}^d \rightarrow \mathbb{R}$
Subset Ranking with Real-Valued Relevance Labels

query 1

query 2

...
Subset Ranking with Real-Valued Relevance Labels

- **Query space** $Q$
- **Document space** $D$
- **Query-document feature mapping** $\phi : Q \times D \rightarrow \mathbb{R}^d$
- **Input**: Training sample $S = (S^1, \ldots, S^m)$:
  
  $$S^i = ((\phi^i_1, y^i_1), \ldots, (\phi^i_{n_i}, y^i_{n_i})) \in (\mathbb{R}^d \times \mathbb{R})^{n_i}$$

  where

  $$\phi^i_j = \phi(q^i, d^i_j), \quad y^i_j = \text{relevance of } d^i_j \text{ to } q^i$$

- **Output**: Ranking function $f : \mathbb{R}^d \rightarrow \mathbb{R}$
RankSVM Applied to IR/Subset Ranking

Standard RankSVM

$$\min_{f \in \mathcal{F}_K} \left[ \left( \frac{1}{\sum_{i=1}^{m} \binom{n_i}{2}} \right) \sum_{i=1}^{m} \sum_{1 \leq j < k \leq n_i} \ell_{\text{hinge}} \left( f, (\phi_j^i, y_j^i), (\phi_k^i, y_k^i) \right) + \frac{\lambda}{2} \| f \|_K^2 \right]$$

$$\ell_{\text{hinge}} \left( f, (\phi_j^i, y_j^i), (\phi_k^i, y_k^i) \right) = \left( 1 - \left( \text{sign}(y_j^i - y_k^i) \cdot (f(\phi_j^i) - f(\phi_k^i)) \right) \right)_+,$$

convex upper bound on

$$1 \left( (y_j^i - y_k^i)(f(\phi_j^i) - f(\phi_k^i)) < 0 \right)$$

[Joachims, 2002]
RankSVM Applied to IR/Subset Ranking

RankSVM with Query Normalization & Relevance Weighting

\[
\min_{f \in F_K} \left[ \frac{1}{m} \sum_{i=1}^{m} \frac{1}{\binom{n_i}{2}} \sum_{1 \leq j < k \leq n_i} \ell_{\text{hinge}}^{\text{rel}}(f, (\phi_j^i, y_j^i), (\phi_k^i, y_k^i)) + \frac{\lambda}{2} \|f\|_K^2 \right]
\]

\[
\ell_{\text{hinge}}^{\text{rel}}(f, (\phi_j^i, y_j^i), (\phi_k^i, y_k^i)) = \left( |y_j^i - y_k^i| - \left( \text{sign}(y_j^i - y_k^i) \cdot (f(\phi_j^i) - f(\phi_k^i)) \right) \right)^+, \]

convex upper bound on

\[
|y_j^i - y_k^i| \mathbb{I} \left( (y_j^i - y_k^i)(f(\phi_j^i) - f(\phi_k^i)) < 0 \right)
\]

[Agarwal & Collins, 2010; also Cao et al, 2006]
Ranking Performance Measures in IR

Mean Average Precision (MAP)

Binary Labels: \( y_j \in \{0, 1\} \)

\[
\text{MAP}_S(f) = \frac{1}{m} \sum_{i=1}^{m} \left[ \frac{1}{\{j : y_j^i = 1\}} \sum_{j : y_j^i = 1} \text{prec}_{r_j^i}(f) \right]
\]

\( r_j^i \) = rank of document \( d_j^i \) for query \( q^i \)

\( \text{prec}_{r}(f) \) = fraction of positives in top \( r \) documents for query \( q^i \)
Ranking Performance Measures in IR

Normalized Discounted Cumulative Gain (NDCG)

General Real-Valued Labels: \( y_j \in \mathbb{R} \)

\[
\text{NDCG}_S(f) = \frac{1}{m} \sum_{i=1}^{m} \left[ \frac{1}{Z_i} \sum_{r=1}^{n_i} \frac{2^{y_i^r} - 1}{\log_2(r + 1)} \right]
\]

\( \pi_r^i \) = index of document ranked at position \( r \) for query \( q^i \)

\( Z_i \) = normalization constant

\[
\text{NDCG}@k_S(f) = \frac{1}{m} \sum_{i=1}^{m} \left[ \frac{1}{Z_i} \sum_{r=1}^{k} \frac{2^{y_i^r} - 1}{\log_2(r + 1)} \right]
\]
Ranking Algorithms for Optimizing MAP/NDCG

- SVMMAP [Yue et al. 2007]
- SVMNDCG [Chapelle et al. 2007]
- LambdaRank [Burges et al. 2007]
- AdaRank [Xu & Li 2007]
- Regression-based algorithm [Cossock & Zhang 2008]
- SoftRank [Taylor et al. 2008]
- SmoothRank [Chapelle & Wu 2010]
## LETOR 3.0/OHSUMED Data Set

[Liu et al, 2007]

<table>
<thead>
<tr>
<th>No. of Queries</th>
<th>Relevance Labels</th>
<th>Total no. of Query-Doc Pairs</th>
<th>Avg. no. of Docs/Query</th>
<th>No. of Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>106</td>
<td>2 : definitely relevant</td>
<td>16,140</td>
<td>152</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>1 : partially relevant</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 : not relevant</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Further Reading & Resources

[Incomplete!]
Early Papers on Ranking


Generalization Bounds for Ranking


Bioinformatics/Drug Discovery Applications

S. Agarwal and S. Sengupta, Ranking genes by relevance to a disease, CSB 2009.

Other Applications

Natural Language Processing


Collaborative Filtering


Manhole Event Prediction

IR Ranking Algorithms

Y. Cao, J. Xu, T.-Y. Liu, H. Li, Y. Hunag, and H.W. Hon, Adapting ranking SVM to document retrieval, SIGIR 2006.


IR Ranking Algorithms


S. Agarwal and M. Collins, Maximum margin ranking algorithms for information retrieval, ECIR 2010.
NIPS Workshop 2005
Learning to Rank

SIGIR Workshops 2007-2009
Learning to Rank for Information Retrieval

NIPS Workshop 2009
Advances in Ranking

American Institute of Mathematics Workshop in Summer 2010
The Mathematics of Ranking
Tutorial Articles & Books

